An LMI Approach to Fuzzy Pole Cluster for Regulating Wind Energy Conversion System

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Abstract- This paper addresses the design of fuzzy state feedback controller that has not only the ability to stabilize the fuzzy model/system but also to control the transient behaviour and closed loop pole location for wind energy conversion systems which present interesting control demands and exhibits intrinsic non-linear characteristics. The proposed fuzzy controller is employed to regulate indirectly the power flow in the DC link by regulating the DC current. First, a Takagi-Sugeno fuzzy model is employed to represent a non-linear system. Then a model-based fuzzy controller design utilizing the concept of parallel-distributed compensation is developed. Additional constraints on the closed loop pole location are satisfied. Satisfactory time response and closed loop damping are achieved by forcing the closed loop poles into a suitable sub-region of the left half plane. These conditions are expressed in terms of linear matrix inequalities (LMI’s), which can be solved very efficiently using convex optimization techniques. The design techniques are applied to a dynamic model of wind energy conversion system to illustrate the feasibility of the proposed solution.

I. NOMENCLATURE

\( i_{d}, i_{q} \) Peak d and q axes stator currents
\( i_{dr}, i_{qr} \) Peak d and q axes rotor currents
\( v_{d}, v_{q} \) Peak d and q axes stator voltages
\( L_{m} \) Magnetizing inductance
\( L_{r}, L_{s} \) Rotor and stator self inductances
\( r_{r}, r_{i} \) Rotor and stator resistances
\( \omega_{r} \) Rotor speed (rad/sec)
\( \omega_{s} \) Electrical frequency (rad/sec)
\( N \) Number of poles
\( i_{dc}, i_{qc} \) Peak d and q axes capacitor currents
\( i_{f} \) Peak magnitude of ac line current flowing into the converter
\( i_{qf} \) Peak d and q axes currents flowing into the converter
\( C \) Self excitation capacitance
\( L_{DC}, R_{DC} \) DC link inductance and resistance
\( V_{G} \) Converter DC voltage
\( V_{RM} \) Maximum converter DC voltage
\( i_{DC} \) DC link current
\( \alpha_{G} \) Converter firing angle \((0 \leq \alpha_{G} \leq 90^\circ)\)
\( \alpha_{I} \) Inverter firing angle \((90^\circ \leq \alpha_{I} \leq 180^\circ)\)
\( B, J \) Friction and inertia of the rotating parts of the system.

II. INTRODUCTION

In the world of today there is a need for alternatives to the large coal and oil fired power plants. Renewable energy is one way to go, and in particular wind turbines have proven to be a solution 0. Among the interdisciplinary factors contributing to the success of the wind generation technology are the arrival of the new power devices technologies, new circuit topologies and novel control strategies. The control of grid connected wind turbines contrasts strongly with that of conventional power stations. Wind energy conversion system control is considered an interesting application area for control theory and engineering because the wind energy conversion system (WECS) dynamics consists of many dynamics integrated or combined together (such as structural dynamics, drive train dynamics, generator dynamics and the dynamics of the control system hardware). These dynamics are lightly damped, they are mainly dependent on wind speed; more particularly theses dynamics have highly non-linear nature. The objectives for the control system are unusual in that it is not the dynamics induced by transient loads, which is required to be minimized, but the fatigue damage induced by them. [2].

Variable speed Wind energy conversion (VSWCS) system is basically composed of wind turbine coupled to electric generator. The electrical generator output can be connected to the grid by means of modern converter system. This configuration is shown in fig.1. Systems with synchronous or asynchronous generator, a rectifier, a DC interconnected circuit and grid commutated inverters allow the turbine rotating speed to be decoupled from the grid frequency. These are becoming more and more important because they reduce the strains on the mechanical parts and also the electrical power variation [3]. Optimizing the performance (i.e. the efficiency) of the main components of variable speed wind turbine is considered the essential objective, which dominates the literature on control of wind energy conversion systems especially those connected to the grid. This optimization process implies two control targets. The first one is maximization the output power below rated wind speed i.e. maximizing the energy capture from the rotor blades. The second one is limitation of the output power above rated wind speed where the goal of the controller is to track the nominal (rated) power of the wind turbine. The first target can be achieved by keeping the tip speed ratio of the turbine at its optimum value despite wind variation. In [4], a speed control problem to achieve this target is discussed. A direct adaptive control strategy is proposed. It is based on the combination of two control actions; a radial basis function network based adaptive controller and a supervisory controller based on crude bounds of the system non-linearities. This technique - in spite of its succession- has main problem that, the
supervisory control action has a discontinuous policy, which is undesirable in practice.

Output power regulation may also be desired even below rated wind speed. For instance, when a power demand has to be tracked such as in some autonomous systems or when the supplied power is restricted by power quality problems of weak grids. In this case, the system is usually operated in an output power regulation mode. In [5], a dead beat control of output power was proposed. The knowledge of wind velocity is needed for controller implementation. However, a precise knowledge of the turbine aerodynamics is needed for estimation purposes. A dynamical sliding mode control for power regulation of VSWECS using induction generators is developed in [7],[8]. However, chattering phenomena that result in low control accuracy and high heat loss in electrical power circuits is inevitable in the sliding mode control. In [9] the power transfer through a DC link for certain WECS configuration is regulated indirectly by regulating DC link current. A simple PI controller is designed using standard frequency domain optimization. Since the PI controller is tuned for a specific linearized model about an operating point, it may provide insufficient damping for other different operating points.

Takagi–Sugeno (T–S) fuzzy modelling allows modelling the non-linear dynamics by means of a suitable “blending” of linear subsystems, each one of them corresponding to different operation points [10]. Then, local controllers can be designed for each subsystem, obtaining the aggregate controller by the same blending rules used to define the overall fuzzy system. The control design is carried out based on the fuzzy model via the so-called parallel-distributed compensation scheme. The idea is that for each local linear model, a linear feedback control is designed. The resulting overall controller, which is non-linear in general, is again a fuzzy blending of each individual linear controller. However, this design procedure guarantees the stability of the fuzzy system but they provide little control over the transient behaviour and closed loop pole location.

The design procedure in this paper aims to design a stable fuzzy controller to guarantee the existence of the closed loop system poles in some predefined zone to ensure satisfactory transient behaviour. More significantly, the stability analysis and control design problems are reduced to linear matrix inequality (LMI) problems [11]. Numerically, the LMI problems can be solved very efficiently by means of powerful tools available to date in the mathematical programming literature. Since LMI’s intrinsically reflect constraints rather than optimality it offers more flexibility for combining several constraints on the closed loop system. Therefore, by solving the stability LMI’s and pole cluster constraint LMI’s simultaneously, the feedback gain, which guarantees global asymptotic stability and satisfy the desired performance are determined.

III. DYNAMIC MODELING OF WECS

In this section the dynamic modelling of each subsystem for the proposed WECS is presented. The objective of the modelling excludes the self-excitation process of the induction generator and is aimed only at the behaviour after self excitation. The WECS adopted in this work is shown in Fig.1. It consists of three bladed horizontal axis wind turbine (HAWT) drives self excited induction generator, which is connected to the grid via AC-DC-AC link scheme. The AC-DC-AC link scheme used in this paper consists of a controlled rectifier, a DC link reactor and a controlled line commutated inverter.

\[
\text{Fig. 1. Wind energy conversion system}
\]

**A. Horizontal Axis Wind Turbine**

A simplified model for HAWT is used for the aerodynamics with average wind speed as input. The torque at the turbine shaft neglecting losses in the drive train is given by [12]:

\[
T_m = 0.5 \pi \rho C_f R^2 V_w^2
\]

where \(T_m\) is the turbine mechanical torque, \(\rho\) is the air density, \(R\) is the turbine radius, \(C_f\) is the turbine torque coefficient and \(V_w\) is the wind velocity.

**B. Self Excited Induction generator**

The non-linear dynamical model of the induction machine in the d-q reference frame for the proposed WECS when generating can be stated as follows:

\[
\begin{align*}
\dot{p}_{id} &= -k_1q_v i_d + (w_i + k_2 L_m w_i) i_d + k_1 L_m w_i i_q + k_2 L_i i_d \left(\frac{(r+L_m k_2 r')/L_m}{i_q} + (L_m k_2 w_r w_q) i_d\right) \\
\dot{p}_{id} &= (w_i + k_2 L_m w_i) i_q + k_1 q_v i_d + k_2 L_m w_i i_q - k_2 r_i q_v i_q - k_1 q_v i_d \left(\frac{(r+L_m k_2 r')/L_m}{i_q} + (L_m k_2 w_r w_q) i_d\right) \\
\dot{p}_{iq} &= k_1 q_v i_q + L_i k_2 w_i i_q - L_i k_2 w_i i_q - L_i k_2 w_i i_q - (r+L_m k_2 r')/L_m + k_1 q_v i_q \\
p_w &= -(B/j) w_v + (3N^2 L_m/8) (i_q i_d - L_m l_q l_d) + (N/2) T_m
\end{align*}
\]

where \(k_1 = L_r (L_a L_r - L_m^2)\) and \(k_2 = L_m (L_a L_m - L_m^2)\).

The above equations were derived assuming that the initial orientation of the q-d synchronously rotating reference frame is such that the d-axis is aligned with stator terminal voltage phasor (i.e. \(v_{eq} = 0\)).

**C. Self excitation Capacitor**

The dynamical equations of self-excitation capacitance can be described as follows (assuming \(V_{eq} = 0\))

\[
\begin{align*}
\dot{p}_{vd} &= \frac{i_d}{C} \\
\dot{w}_c &= \frac{i_q}{CV_{ds}}
\end{align*}
\]

where the capacitor current \(i_c\) and \(i_q\) flow as shown in Fig. 2.
D. Asynchronous AC-DC-AC Link

The AC-DC-AC link scheme used in this paper consists of a controlled rectifier, a DC link reactor and a controlled line commutated inverter. Assuming that the converter is lossless, the instantaneous power balance equation is written as

\[ R_{DC} I_D i_d + V_{DC} - V_{qs} = 0 \]

where \( V_{qs} \) is the output voltage of the inverter. The ac and dc currents of the converter are related by

\[ i_d = \frac{2\sqrt{3}}{\pi} I_{DC} \cos \alpha_R \]

\[ i_q = -\frac{2\sqrt{3}}{\pi} I_{DC} \sin \alpha_R \]

The peak d and q axes currents flowing into converter can be deduced with the assumption of lossless converter

\[ i_d = \sqrt{3} I_{DC} \]

\[ i_q = 0 \]

The dynamics introduced by the DC link is expressed as

\[ L_{DC} i_d + R_{DC} i_d = V_{DC} - V_I \]

The detailed of these above equation and its parameters can be found in [9].

IV. PROPOSED CONTROL STRUCTURE

A. TS Fuzzy Model

The overall fuzzy model of a system is achieved by fuzzy "blending" of the linear system models. The general T-S fuzzy system is of the following form:

**Plant Rule i:**

If \( x_i(t) = F_{ij} \) and \( \ldots \) and \( x_n(t) = F_{in} \)

Then \( \dot{x}(t) = A_i x(t) + B_i u(t) \) for \( i = 1, \ldots, r \)

where \( x(t) = [x_1(t), x_2(t), \ldots, x_n(t)]^T \in \mathbb{R}^n \) denotes the state vector, \( u(t) = [u_1(t), u_2(t), \ldots, u_m(t)]^T \in \mathbb{R}^m \) denotes the control input, \( F_{ij} \) is the fuzzy set, \( A_i \in \mathbb{R}^{m \times n}, B_i \in \mathbb{R}^{n \times m}, C_i \in \mathbb{R}^{p \times n} \) and \( r \) is the number of IF-THEN rules.

Given a pair of \( (x(t), u(t)) \), the final output of the fuzzy system is inferred as follows [10]

\[ \dot{x}(t) = \sum_{i=1}^{r} h_i(x(t))(A_i x(t) + B_i u(t)) \]

Where

\[ \mu_i(x(t)) = \prod_{j=1}^{g} F_{ij}(x(t)) \]

\[ h_i(x(t)) = \frac{\mu_i(x(t))}{\sum_{i=1}^{r} \mu_i(x(t))} \]

\[ F_{ij}(x(t)) \] is the grade of membership of \( x_i(t) \) in \( F_{ij} \).

In this paper, we assume

\[ \mu_i(x(t)) \geq 0, \quad \text{for} \quad i = 1, 2, \ldots, r \]

and \( \sum_{i=1}^{r} \mu_i(x(t)) > 0 \) for all \( t \). Therefore, we get

\[ h_i(x(t)) \geq 0 \quad \text{for} \quad i = 1, 2, \ldots, r \]

and

\[ \sum_{i=1}^{r} h_i(x(t)) = 1 \]

B. Parallel distributed Compensation Technique

The concept of parallel-distributed compensation (PDC) in [13] is utilized to design fuzzy controllers to stabilize fuzzy system (17). The resulting overall fuzzy controller, which is non-linear in general, is a fuzzy blending of each individual linear controller. The fuzzy controller shares the same fuzzy sets with the fuzzy system (17).

**Control rule j:**

If \( x_i(t) = F_{ij} \) and \( \ldots \) and \( x_n(t) = F_{in} \)

Then \( u(t) = -K_j x(t) \) for \( j = 1, 2, \ldots, r \)

Hence, the fuzzy control decision is given by:

\[ u(t) = -\sum_{j=1}^{r} h_j(x(t))K_j x(t) \]

where \( K_j \) (for \( j = 1, 2, \ldots, r \)) are the controller gain. Note that the controller (22) is non-linear in general.

C. Stability Analysis via Lyapunov Approach

A sufficient stability condition derived in [14] for ensuring stability of (17) is given as follows:

**Theorem 1:** The equilibrium of a fuzzy control system (17) is asymptotically stable in the large via PDC (22) if
there exists a common positive definite matrix $P$ such that for $i,j = 1,2,\ldots,r$

$$(A_i - B_iK_j)^T P_p + P (A_i - B_iK_j) < 0$$ (21)

The proof can be found in [14]. Its basic idea is the use of a quadratic Lyapunov function

$V(x) = x^T P x$. Theorem 1 thus presents a sufficient condition for quadratic stability of the system (17) using the control law (22).

D. Lyapunov Condition For Pole Clustering

The LMI-based characterization for a wide class of pole clustering regions as well as an extended Lyapunov Theorem for such regions is presented in this section. It is known that the transient response of a linear system is formulated. The most important work of the proposed fuzzy stability condition constrained by pole cluster condition is known that the transient response of a linear system is

Theorem for such regions is presented in this section. It is clustering regions as well as an extended Lyapunov

region, a satisfactory transient response can be ensured.

Regions that can be handled by LMI include $\alpha$-stability regions $Re(s) \leq -\alpha$ vertical strips, disks, conic sectors, etc. Another interesting region for control purposes is the set $S (\alpha, r, \theta)$ of complex numbers $x + j y$ such that $x (-\alpha \langle 0, [x + j y] \rangle \langle r, \tan \theta \langle -|y| \rangle$ as shown in Fig. 3. Confining the closed-loop poles to this region ensures a minimum decay rate $\alpha$, a minimum damping ratio $\zeta = \cos \theta$, and a maximum un-damped natural frequency $w_0 = r \sin \theta$. This in turn bounds the maximum overshoot, the frequency of oscillatory modes, the delay time, the rise time, and the settling time [15].

The concept of LMI region [16] is useful to formulate pole cluster objectives in LMI terms. LMI region are convex subsets $D$ of the complex plane characterized by

$$D = \{ z \in C : f_D(z) < 0 \}$$ (22)

with

$$f_D := \alpha + z \beta + \bar{z} \beta^T = [\alpha_{kl} + \beta_{kl} z + \bar{\beta}_{kl} \bar{z}]_{lsk,lsm}$$ (23)

where $\alpha = [\alpha_{kl}] \in \mathbb{R}^{n \times n}$ and a matrix $\beta = [\beta_{kl}] \in \mathbb{R}^{n \times n}$. The matrix-valued function $f_D$ is called the characteristic function of the region $D$.

Let $D$ be a sub-region of the complex left-half plane. A dynamical system $\dot{x} = Ax$ is called $D$-stable if all its poles lie in $D$ (that is, all eigen values of the matrix $A$ lie in $D$). By extension, $A$ is then called $D$-stable.

It was shown in [16] that the pole cluster constraint is satisfied if and only if there exists

$$[\alpha_{kl} X + \beta_{kl} A_k X + \bar{\beta}_{kl} \bar{X}_k]_{lsk,lsm} < 0$$ (24)

where $A_k = A_i - B_iK_j$ in our fuzzy case and $X$ positive definite symmetric matrix.

By this LMI formulation, a necessary and sufficient stability condition constrained by pole cluster condition is formulated. The most important work of the proposed fuzzy control problem is how to solve the common solution $X = X^T \geq 0$. The common $X$ problem can be solved efficiently via convex optimization techniques for linear matrix inequality (LMI’s).

The condition in (26) is not yet convex because of the products $K_jX$, arising in terms like $A_iX$. However, convexity is readily restored by rewriting in terms of $X, Y_i$ where $Y_i = KX$ [11]. This simple change of variable leads to the following sub optimal LMI stability condition with pole assignment in arbitrary LMI regions.

$$\left[ \alpha_{kl} X + \beta_{kl} (A_iX + B_i Y_j) + \bar{\beta}_{kl} (A_iX + B_i Y_j)^T \right]_{lsk,lsm} < 0$$

$$X > 0.$$ (25)

The inequality in (25) is linear matrix inequality feasibility problem (LMIIP) in $X$ and $Y_i$ which can be solved very efficiently by the convex optimization technique such as interior point algorithm. Software packages such as LMI optimization toolbox in MatLab [17] have been developed for this purpose and can be employed to easily solve the LMIIP.

VI. WECS SIMULATION EXAMPLE

To illustrate the proposed fuzzy control approach, the WECS shown in Fig.1 is considered. The control objective in this example is to design a fuzzy state feedback controller that has not only the ability to stabilize the fuzzy model/system but also to control the transient behaviour and closed loop pole location for wind energy conversion system. The proposed fuzzy controller is employed to regulate indirectly the power flow in the DC link by regulating the DC current.

To minimize the design effort and complexity, the number of fuzzy rules should be small. The Takagi-Sugeno fuzzy model that approximates the dynamics of the WECS non-linear plant ((2) to (6)) can be represented by the following two-rule fuzzy model:

Rule 1: IF $i_{dc}$ is about 2.5A
THEN $\dot{x} = A_1 x + B_1 u$

Rule 2: IF $i_{dc}$ is about 3.2A
THEN $\dot{x} = A_2 x + B_2 u$

where $x_1, x_2, x_3, x_4, x_5, x_6$ and $x_7$ denotes $i_{qs}, i_{ds}$

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The control action are \( u_1 \) and \( u_2 \) denoting \( \alpha_R \) and \( \alpha_f \). The matrices \( A_i \) and \( B_i \) can be obtained by linearizing the non-linear system in (2) to (6) around some suitable points. In this example, we use MatLab/Simulink software package to get the suitable operating points and the corresponding linearized systems. The LMI region chosen in this example is shown in Fig. 3 with \( \alpha = -0.5 \) and \( \theta = 45^\circ \). Using (21), (25) and the LMI optimization toolbox in MatLab [17], we obtain the common solution for \( P \) as follows:

\[
P = \begin{bmatrix}
1.0638 & -0.1819 & 1.0176 & -0.2052 & -0.0197 & -0.0611 & -0.0085 \\
-0.1819 & 0.6387 & 1.3166 & 0.6577 & -0.0163 & -0.0373 & 0.1038 \\
1.0176 & -0.1366 & 1.0036 & -0.1548 & -0.0192 & -0.0548 & -0.0064 \\
-0.2052 & 0.6577 & -0.1548 & 0.7235 & 0.0018 & -0.0269 & 0.1122 \\
-0.0197 & -0.0163 & -0.0192 & 0.0018 & 0.0328 & 0.0066 & -0.0041 \\
-0.0611 & -0.0373 & -0.0548 & -0.0269 & 0.0066 & 0.0273 & -0.0095 \\
-0.0085 & 0.1038 & -0.0064 & 0.1122 & -0.0041 & -0.0095 & 0.0286
\end{bmatrix} \times 10^5
\]

The controller parameters are found to be

\[
K_1 = \begin{bmatrix}
2.7336 & -1.6168 & 2.3533 & -1.6472 & -0.1869 & -0.8886 & 0.2668 \\
-6.1684 & 52.6068 & -7.2361 & 58.6127 & -1.4349 & -2.6538 & 15.1083
\end{bmatrix}
\]

\[
K_2 = \begin{bmatrix}
3.6039 & -2.0501 & 3.0276 & -2.0299 & -0.2196 & -1.0934 & 0.5149 \\
\end{bmatrix}
\]

The controller parameters without pole cluster constraint are found to be

\[
K_1 = \begin{bmatrix}
1.3586 & -1.6243 & 1.5315 & -1.5652 & -0.0016 & 0.0161 & -0.2479 \\
47.2568 & -437.6046 & 88.4233 & -418.7878 & -6.9963 & 12.3979 & -1.0768
\end{bmatrix}
\]

\[
K_2 = \begin{bmatrix}
0.5898 & -0.9800 & 0.7010 & -0.9800 & 0.0090 & 0.0058 & -0.2443 \\
\end{bmatrix}
\]

Figures 4-7 present the simulation results for the proposed model based fuzzy control system. Fig. 4 and 6 show the trajectory of \( x_7 \) (IDC) for fuzzy PDC controller without and with pole cluster constraints. The response of the fuzzy system with pole cluster constraint is well damped, faster and has smaller over shoot compared to the response of the fuzzy system without pole cluster constraint. Fig. 5 and 7 show the control actions without and with pole cluster constraint. It is clear that no excessive fast change in control input commands with pole cluster constraint are shown. From the simulation results, the performance of the proposed fuzzy pole cluster controller is better than that fuzzy controller only.

V. CONCLUSION

In this paper, a model based fuzzy control scheme is proposed to cope with the intrinsic non-linear behaviour of WECS. The Takagi Sugeno fuzzy model is employed to approximate the non-linear model of WECS. Based on the fuzzy model, a fuzzy controller is developed to guarantee not only the stability of fuzzy model and fuzzy control system for the WECS but also control the transient behaviour of the system. Satisfactory time response and closed loop damping are achieved by forcing the closed loop poles into a suitable sub-region of the left half plane. The design procedure is conceptually simple and natural. Moreover, the stability analysis and control design problems are reduced to LMI problems. Therefore, they can be solved very efficiently in practice by convex programming techniques for LMI's.
VI. REFERENCE


